Note Title 19-10-2016

Commutation and Uncertainty Principle

$$[\hat{A}, \hat{B}] = 0 = [\hat{A}\hat{B} - \hat{B}\hat{A}]$$

2 operators commute

$\Rightarrow$ commute $[\hat{T}_x, \hat{p}_x] \Rightarrow (\hat{T}_x \hat{p}_x - \hat{p}_x \hat{T}_x) f(x)$

equal

$$\hat{T}_x \hat{p}_x f(x) = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - i\hbar \frac{d}{dx} f$$

$$= \underline{\frac{i\hbar^3}{2m}\frac{d^3 f}{dx^3}}$$

$$\hat{p}_x \hat{T}_x f(x) = -i\hbar \frac{d}{dx} - \frac{\hbar^2}{2m}\frac{d^2}{dx^2} f$$

$$= \underline{\frac{i\hbar^3}{2m}\frac{d^3 f}{dx^3}}$$

$$[\hat{x}, p_x] \Rightarrow [\hat{x}\hat{p}_x - \hat{p}_x \hat{x}] f(x)$$

$$\hat{x}\hat{p}_x f(x) = x - i\hbar \frac{d}{dx} f = -i\hbar x \frac{df}{dx}$$

$$\hat{p}_x \hat{x} f(x) = -i\hbar \frac{d}{dx}[x f(x)]$$

$$= -i\hbar f(x) - i\hbar x \frac{df}{dx}$$

$[\vec{x}, p_x] = i\hbar$

Variance :

$$\sigma_A^2 = \langle (A - \langle A \rangle)^2 \rangle$$

$$\Downarrow$$

$$\int \Psi^* (\hat{A} - \langle A \rangle)^2 \Psi \, dx$$

$$= \int \Psi^* (\hat{A}^2 - 2\hat{A}\langle A \rangle + \langle A \rangle^2) \Psi \, dx$$

$$= \int \Psi^* \hat{A}^2 \Psi \, dx - 2\int \Psi^* \hat{A} \langle A \rangle \Psi \, dx + \int \Psi^* \langle A \rangle^2 \Psi \, dx$$

$$= \langle A^2 \rangle - 2\langle A \rangle \langle A \rangle + \langle A \rangle^2$$

$$\sigma_A^2 = \langle A^2 \rangle - \langle A \rangle^2$$

$$\sigma_A \sigma_B \geq \frac{1}{2} \left| \int \Psi^* [\hat{A}, \hat{B}] \Psi \, dx \right|$$

$$\sigma_A \sigma_B \geq \frac{1}{2} \left| \langle [\vec{A}, \hat{B}] \rangle \right|$$

<u>Particle in a box (1-D)</u>

$$\Delta x \, \Delta p_x \quad \text{or} \quad \sigma_x \, \sigma_{p_x}$$

$$\Delta x \text{ or } \sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

$$\Delta p_x \text{ or } \sigma_{p_x} = \sqrt{\langle p_x^2 \rangle - \langle p_x \rangle^2}$$

$$\sigma_x = \sqrt{\left(\frac{L^2}{3} - \frac{L^2}{2n^2\pi^2} \right) - \left(\frac{L}{2} \right)^2}$$

$$= \sqrt{\frac{L^2}{12} - \frac{L^2}{2n^2\pi^2}}$$

$$= \sqrt{\frac{L^2}{4n^2\pi^2} \left[\frac{n^2\pi^2}{3} - 2 \right]}$$

$$\underline{\sigma_x = \frac{L}{2n\pi} \sqrt{\left(\frac{n^2\pi^2}{3} - 2 \right)}}$$

$$\sigma_{p_x} = \sqrt{\langle p_x^2 \rangle - \underbrace{\langle p_x \rangle^2}_{\to 0}}$$

$$= \sqrt{2m \langle E \rangle} = \sqrt{2m \frac{n^2 h^2}{8mL^2}}$$

$$\sigma_{p_x} = \sqrt{\frac{n^2 h^2}{4L^2}} = \frac{nh}{2L}$$

$$\sigma_x \sigma_{p_x} = \frac{nh}{2L} \frac{L}{2n\pi} \sqrt{\left(\frac{n^2\pi^2}{3} - 2\right)}$$

$$= \frac{h}{4\pi} \sqrt{\left(\frac{n^2\pi^2}{3} - 2\right)}$$

$\underline{\underline{n=1}}$ $\qquad \sigma_x \sigma_{p_x} = 0.568\hbar$

$$\sigma_x \sigma_{p_x} \geq \frac{\hbar}{2}$$

$$\sigma_A \sigma_B \geq \frac{1}{2} \left| \langle [\hat{A}, \hat{B}] \rangle \right|$$

$$\hat{A} = x; \quad \hat{B} = p_x$$

$$\langle [\hat{A}, \hat{B}] \rangle = \int \psi^* [\hat{x}, \hat{p_x}] \psi \, dx$$

$$= \int \psi^* [\hat{x}\hat{p_x} - \hat{p_x}\hat{x}] \psi \, dx$$

$$= i\hbar \int \psi^* \psi \, dx$$

$$= i\hbar$$

$$\sigma_A \sigma_B \geq \frac{1}{2} |i\hbar|$$

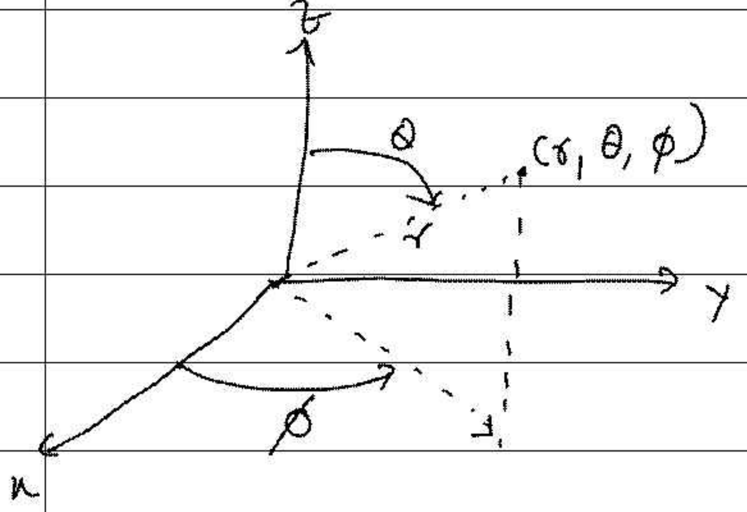
$$\sigma_x \sigma_{p_x} \geq \frac{\hbar}{2}$$

Rotational Motion

$0 < r < \infty$

$0 < \theta < \pi$

$0 < \phi < 2\pi$



$x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$z = r \cos\theta$

spherical polar coordinates

$$\hat{H} = -\frac{\hbar^2}{2m} \underbrace{\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right]}_{\nabla^2}$$

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2}{\partial \phi^2}$$

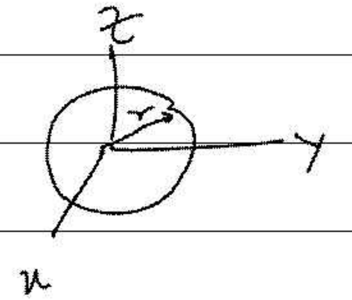
$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\Lambda^2 \quad \cdots \quad (1)$$

$$\Lambda^2 = \frac{1}{\sin\theta} \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial \phi^2}$$

$\uparrow$

Legendrian

Particle on a ring



r is fixed